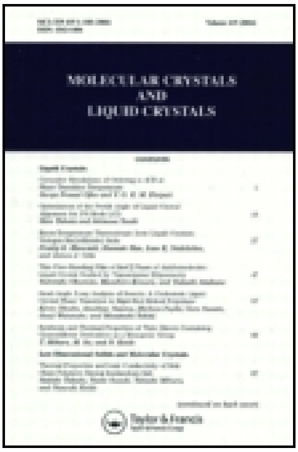

Molecular Crystals and Liquid Crystals Science and Technology. Section A. Molecular Crystals and Liquid Crystals

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl19

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Version of record first published: 23 Sep 2006.

Mol. Cryst. Liq. Cryst. 1995, Vol. 262, pp. 629–643
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SELF-FOCUSING AND SELF-TRAPPING IN SMECTIC A LIQUID CRYSTAL

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Abstract The self-focusing of a laser beam due to the Kerr nonlinearity caused by layer deformations in a smectic A liquid crystal (SmA) is considered. It is shown that for a sufficiently high intensity the beam undergoes self-trapping forming a spatial soliton. The same mechanism is responsible for the self-trapping of a surface-guided light wave at the interface of a linear medium and SmA. Numerical estimations show that the new Kerr nonlinearity is approximately 10 times greater than a nonlinearity caused by bulk compression. Self-trapping effect may be observed in smectic A liquid crystal samples of a thickness of 0.01 cm and at a light beam power of an order of magnitude of 0.002 MW.

1. INTRODUCTION

The propagation of a laser beam through a nonlinear optical material is accompanied by the self-focusing and self-trapping effects due to the dependence of the refractive index n_r of the medium on the light intensity.[1,2] In highly nonlinear media, such as liquid crystals, these self-effects are strongly manifested.[2–4] Nematic liquid crystals (NLC) and smectic A liquid crystals (SmA) possess the inversion symmetry,[5] and their refractive indices display the quadratic dependence on the electric field E of a light beam,[2–4] thus being Kerr media.[6] In NLC the giant optical nonlinearity (GON)[4] is determined by the field-induced director reorientation.[2–4,7] The self-focusing in NLC determined by GON have been thoroughly investigated both theoretically and experimentally.[2–4,7] The nonlinear part n_2 of the refractive index n_r appeared to be of an order of magnitude of 10^{-3} esu, while for a typical nonlinear medium

$$n_2 \cong 10^{-11}\text{esu}$$

in the case of molecular-orientation Kerr effect.[1,2] The magnitude of GON is determined by a low energy of a director orientation in NLC $K \cdot w^{-2}$, where $K \sim (10^{-7} - 10^{-6})$ dyn is a Frank elastic constant,[5], and w is a spatial dimension of a perturbation in NLC. As a result, the self-focusing in NLC caused by GON can occur only in the case of a low light intensity.

The self-focusing of an extraordinary beam in SmA has been considered earlier for a very special case, when the normal deformation of smectic layers was ignored.[8] It was assumed that the initial homogeneous state of the sufficiently small sample of SmA has been supported by an external magnetic field H, while the contribution of both elastic and orientational terms into the free energy density has been neglected.

It has been shown that in this case the strong nonlinearity similar to the one in NLC would occur, if the light intensity P is sufficiently low: ~ 200 W/cm^2.

The validity of this approach is limited at least by two factors :

1. The intensity of the incident light wave must be small in comparison with the anisotropic part of the external magnetic field energy density;
2. The thickness of a SmA sample must be much smaller than a light beam width.[8]

Actually, SmA in the absence of a layer compression behaves like NLC representing a two-dimensional liquid.[5]

In this paper a new mechanism of the self-focusing and self-trapping in SmA caused by the normal layer deformation is considered. The elastic constant $B \sim 10^8$ erg/cm^3 associated with these deformations is large in comparison with the reorientation energy, but it is much less than the elastic constant associated with a bulk compression.[5] It is shown that the system of compressed layers behaves as a self-focusing medium with respect to an ordinary and an extraordinary light beams. The nonlinear part n_2 of the refractive index of SmA is at least of two orders of magnitude greater than the one for an electrostriction nonlinearity in organic liquids.[1] It is shown that the self-trapping[1] of ordinary and extraordinary light beams in a form of a spatial soliton[9] is possible. The self-trapping of an extraordinary beam occurs only when the anisotropy angle is sufficiently small.

The width of a spatial soliton and a critical light power are calculated. The analysis is based on the solution of the self-consistent system of the wave equation for a nonlinear inhomogeneous anisotropic medium and the equation of motion for the medium in the presence of an external field.[6]

In the section 2 of the article the reduced equations[6] for both an ordinary and an extraordinary light beams propagating in SmA are obtained, the possibility of the self-focusing is shown, and the nonlinear part n_2 of the refractive index is evaluated. In the section 3 the solutions corresponding to the self-trapping effect are obtained. The conclusions are presented in the section 4.

2. THE SELF-FOCUSING OF ORDINARY AND EXTRAORDINARY LIGHT BEAMS IN SmA.

The dielectric constant tensor ε_{ik} of SmA including the terms dependent on the layer deformation has the form:[5,10]

$$\begin{gathered}
\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{\perp} + a_{\perp}\frac{\partial u}{\partial z}, \qquad \varepsilon_{zz} = \varepsilon_{\parallel} + a_{\parallel}\frac{\partial u}{\partial z} \\
\varepsilon_{xz} = \varepsilon_{zx} = -\varepsilon_a \frac{\partial u}{\partial x}, \qquad \varepsilon_{zy} = \varepsilon_{yz} = -\varepsilon_a \frac{\partial u}{\partial y} \\
\varepsilon_a = \varepsilon_{\parallel} - \varepsilon_{\perp}
\end{gathered} \tag{1}$$

where $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ are the principal values of the tensor along and normal to the optical axis, and $u(\vec{r}, t)$ is a normal layer displacement along the Z axis. The Z axis is chosen to coincide with the optical axis, and the X and Y axes are chosen to be in the plane of a layer.

It is known that in anisotropic media a propagation of ordinary and extraordinary waves is possible:[11]

$$\vec{E}_{o,e} = \vec{e}^{\,o,e}\left\{A_{o,e}\exp i\left(\vec{k}_{o,e}\vec{r} - \omega t\right) + c.c.\right\} \tag{2}$$

where $\vec{e}^{\,o,e}$, $\vec{k}_0$, $\vec{k}_e$, $A_{o,e}$ are the polarization unit vectors, the wave vectors and the amplitude of the ordinary and extraordinary waves, respectively, and ω is a frequency.

The waves have the following dispersion relations:[11]

$$(k_o)^2 = (\omega/c)^2\,\varepsilon_{\perp} \tag{3a}$$

$$\frac{(k_{ex})^2}{\varepsilon_{\parallel}} + \frac{(k_{ez})^2}{\varepsilon_{\perp}} = \left(\frac{\omega}{c}\right)^2 \tag{3b}$$

where c is the vacuum light velocity.

In general case in a uniaxial crystal an ordinary beam is polarized normal to the incidence plane, and an extraordinary beam is polarized in the incidence plane.[11] In the case of SmA due to its symmetry D_{∞} [5] the coordinate system may be chosen in such a way that an ordinary wave is polarized along the Y axis, and an extraordinary wave polarization vector lays in the XZ plane:

$$\vec{e}^{\,o} = (0, 1, 0) \tag{4a}$$

$$\vec{e}^{\,e} = (e_x^e, 0, e_z^e) \tag{4b}$$

The further analysis is based on the self-consistent system of the wave equation for the light beam propagating in the anisotropic nonlinear inhomogeneous medium and the equation of motion for the nonlinear medium in the presence of the light beam electric field.[6] The wave equation has the form[6,11]

$$\mathrm{rot}\,\mathrm{rot}\vec{E} + \frac{1}{c^2}\frac{\partial^2 \vec{D}^{\,L}}{\partial t^2} = -\frac{1}{c^2}\frac{\partial^2 \vec{D}^{\,N}}{\partial t^2} \tag{5}$$

where $\vec{E}$ is the total electric field in the medium, $\vec{D}^{\,L}$ and $\vec{D}^{\,N}$ are the linear and nonlinear parts of the electric induction, respectively. Combining (1) and (4a, b) we obtain:

$$\begin{aligned}
D_y^L &= \varepsilon_\perp E_o, \quad \vec{D}_\parallel^L = \left\{e_x^e \varepsilon_\perp \vec{x} + e_z^e \varepsilon_\parallel \vec{z}\right\} A_e \exp i\left(\vec{k}_e \vec{r} - \omega t\right) + c.c. \\
D_y^N &= a_\perp \frac{\partial u}{\partial z} E_o, \\
D_x^N &= \left\{e_x^e a_\perp \frac{\partial u}{\partial z} - e_z^e \varepsilon_a \frac{\partial u}{\partial x}\right\} A_e \exp i\left(\vec{k}_e \vec{r} - \omega t\right) + c.c. \\
D_z^N &= \left\{e_z^e a_\parallel \frac{\partial u}{\partial z} - e_x^e \varepsilon_a \frac{\partial u}{\partial x}\right\} A_e \exp i\left(\vec{k}_e \vec{r} - \omega t\right) + c.c.
\end{aligned} \tag{6}$$

Substituting the equations (2) and (6) into the equation (5) and taking into account that an ordinary wave is transverse[11]

$$\mathrm{div}\vec{E}_o = 0$$

we obtain the equations for the ordinary and extraordinary waves:

$$-\nabla^2 E_o = (\omega/c)^2 \left(\varepsilon_\perp + a_\perp \frac{\partial u}{\partial z}\right) E_0 \tag{7a}$$

$$\operatorname{grad}\left(\operatorname{div}\vec{E}_e\right) - \nabla^2\vec{E}_e = (\omega/c)^2 \times$$
$$\times\left\{\left(a_\perp\frac{\partial u}{\partial z}E_{ex} - \varepsilon_a\frac{\partial u}{\partial x}E_{ez}\right)\vec{x} + \left(a_\parallel\frac{\partial u}{\partial z}E_{ez} - \varepsilon_a\frac{\partial u}{\partial x}E_{ex}\right)\vec{z}\right\} \tag{7b}$$

Both amplitudes are assumed to be independent on y which corresponds to the slab-shaped beams[12] with the dimension in the Y direction much greater than in the incidence plane:

$$\frac{\partial A_{o,e}}{\partial y} \equiv 0$$

The free energy density $\mathcal{F}$ of SmA in the presence of an external electric field has the form:[5,11]

$$\mathcal{F} = \frac{1}{2}B\left(\frac{\partial u}{\partial z}\right)^2 - (1/8\pi)\,\varepsilon_{ik}E_iE_k \tag{8}$$

where the small orientational terms are neglected. Minimization of the relationship (8) in respect to the layer displacement $u(\vec{r})$ together with the expressions (1), (2) and (4a, b) yields:

$$B\frac{\partial^2 u}{\partial z^2} = (1/4\pi)\times$$
$$\times\left\{\left[\left(a_\parallel(e_z^e)^2 + a_\perp(e_x^2)^2\right)\frac{\partial}{\partial z} - 2\varepsilon_a e_x^e e_z^e\frac{\partial}{\partial x}\right]|A_e|^2 + a_\perp\frac{\partial|A_o|^2}{\partial z}\right\} \tag{9}$$

The simultaneous analysis of the equations (7a), (7b) and (9) in the case of an arbitrary polarized light wave containing both components is too involved because each amplitude A depends on the different coordinates. However, as it is shown below, in the practically important situation the beams can be assumed to propagate independently in the different directions due to the large optical anisotropy of SmA, and therefore the layer deformations caused by each component are spatially separated. We analyze two particular cases, when only one type of the polarization exists and obtain the condition of the validity of such an approach.

Consider firstly the situation when only the ordinary beam propagates and

$$|A_e| = 0 \tag{10}$$

It is useful to introduce the new coordinates (x', z') parallel and normal to the light beam direction, respectively:

$$\begin{aligned} x' &= \left(\vec{k}_o \vec{r}\right)/k_o = x\sin\theta_o + z\cos\theta_o \\ z' &= -x\cos\theta_o + z\sin\theta_o \end{aligned} \tag{11}$$

where θ_o is the angle between $\vec{k}_o$ and the Z axis.

We are interested in the spatially localized solutions of the type[12]

$$\lim_{z'\to\infty} |A_o(z')| = 0, \qquad \frac{\partial |A_o|}{\partial z'}|_{z'=0} = 0 \\ |A_o(0)| = |A_{o\,max}| \tag{12}$$

Under the conditions (10) and (12) the equation (9) yields:

$$\frac{\partial u}{\partial z} = \frac{a_\perp}{4\pi B}|A_o|^2 \tag{13}$$

Assuming the amplitude A_o to be slowly varying[6] in the direction of propagation

$$\left|\frac{\partial^2 A_o}{\partial x'^2}\right| << \left|k_o \frac{\partial A_o}{\partial x'}\right| \sim \left|\frac{\partial^2 A_o}{\partial z'^2}\right| \tag{14}$$

and combining the equations (2), (3a), (4a), (7a) and (13) we obtain the reduced equation for the amplitude of the ordinary beam. It has the form:

$$i\frac{\partial A_o}{\partial x'} + \frac{1}{2k_o}\frac{\partial^2 A_o}{\partial z'^2} + \left(\frac{\omega}{c}\right)^2 \frac{a_\perp^2}{8\pi B k_o} \times |A_o|^2 A_o = 0 \tag{15}$$

The equation (15) is the nonlinear Schrödinger equation (NSE).[13] The coefficient of the last term in the left-hand side of (15) is positive definite

$$\left(\frac{\omega}{c}\right)^2 \frac{a_\perp^2}{8\pi B k_o} > 0 \tag{16}$$

which corresponds to the stationary two-dimensional self-focusing of the light beam.[11,13] The ordinary beam behaves in SmA as in an isotropic medium, and the refractive index n_r^o may be written as follows:[1,11]

$$n_r^o = n_o + n_2^o |A_o|^2$$

which gives

$$n_2^o = \frac{a_\perp^2}{8\pi B\sqrt{\varepsilon_\perp}}$$

Using the typical values of the material parameters $\sqrt{\varepsilon_\perp} = 1.5$,[2] $B \sim 10^8$ erg/cm^3,[5,14] $a_\perp \sim 1$[15] we obtain $n_2^o \sim 3 \cdot 10^{-10}$ esu which is one or two orders of magnitude greater than the typical values of a nonlinear part of refractive indices for the orientational Kerr effect and electrostriction in ordinary liquids.[1,12]

Consider now the propagation of the extraordinary beam, when

$$|A_o| = 0$$

It has been shown that the diffraction of an extraordinary beam in a medium with a weak optical anisotropy is in general similar to the diffraction of an ordinary one.[16] In liquid crystals the optical anisotropy is much greater than in ordinary optical materials,[2,17] which results in the strong dependence of the self-focusing characteristics on the incidence angle of an extraordinary beam. An extraordinary beam propagates along the direction of the beam vector $\vec{s} \perp \vec{e}^e$, which is determined by the angle θ_e:[11]

$$\theta_e = \arctan\left\{\frac{\varepsilon_\perp}{\varepsilon_\parallel}\tan\theta_1\right\} \tag{17}$$

where θ_1 is the angle between $\vec{k}_e$ and the Z axis. We introduce the coordinates (x'', z'') which are parallel and normal to the beam vector $\vec{s}$, respectively:

$$x'' = x\sin\theta_e + z\cos\theta_e, \quad z'' = -x\cos\theta_e + z\sin\theta_e \tag{18}$$

Applying to the extraordinary beam the conditions of slowly varying amplitude approximation[6] and of the spatial localization[12] analogous to the relationships (12) and (14) we find from (9):

$$\begin{aligned} \frac{\partial u}{\partial z} &= (1/4\pi B)h_e|A_e|^2/\sin\theta_e \\ h_e &= \left(a_\perp(e_x^e)^2 + a_\parallel(e_z^e)^2\right)\sin\theta_e + 2\varepsilon_a e_x^e e_z^e\cos\theta_e \end{aligned} \tag{19}$$

In order to obtain the reduced equation for the extraordinary beam amplitude A_e we must take into account the condition[11]

$$\mathrm{div}\vec{D} = 0 \tag{20}$$

which is not met automatically in this case unlike the case of the ordinary beam. The condition (20) yields

$$\mathrm{div}\vec{E}_e = -\frac{\varepsilon_a}{\varepsilon_\perp}\frac{\partial E_{ez}}{\partial z} \tag{21}$$

Substituting the relationships (2), (3b), (4b), (18), (19), and (21) into the wave equation (6b) we find:

$$
\begin{aligned}
& i\frac{k_{e\perp}}{k_{e\parallel}}\frac{\partial A_e}{\partial z''}\left\{1+\frac{\varepsilon_a}{2\varepsilon_\perp}\left(2\sin\theta_e+\frac{k_{e\parallel}}{k_{e\perp}}\cos\theta_e\right)e_z^e\right\}+ \\
& +i\frac{\partial A_e}{\partial x''}+\frac{1}{2k_{e\parallel}}\frac{\partial^2 A_e}{\partial z''^2}\left(1+\frac{\varepsilon_a}{\varepsilon_\perp}e_z^e\sin\theta_e\right)+ \\
& +\frac{1}{2k_{e\parallel}}\left(\frac{\omega}{c}\right)^2\frac{h_e^2|A_e|^2}{4\pi B\sin^2\theta_e}A_o=0
\end{aligned}
\tag{22}
$$

where $k_{e\perp}$ and $k_{e\parallel}$ are the components of the wave vector which are normal and parallel to the beam vector, respectively. Note that $\theta_e \neq 0$ for the extraordinary beam.

It is seen from the equation (22) that the term with the first derivative on the transverse coordinate z'' emerges due to the existence of the wave vector component $k_{e\perp}$ normal to the beam vector which is typical for extraordinary beams. In the linear approximation the diffraction of an extraordinary beam is similar to the one of an ordinary beam if this term is small in comparison with the second term containing the first derivative on the longitudinal coordinate x'' and therefore may be neglected:

$$
\left|\frac{k_{e\perp}}{k_{e\parallel}}\frac{\partial A_e}{\partial z''}\right| << \left|\frac{\partial A_e}{\partial x''}\right| \tag{23}
$$

Using the relationship (17) and taking into account that

$$
\frac{\partial A_e}{\partial x''}\sim\frac{A_e}{k_e w^2}, \qquad \frac{\partial A_e}{\partial z''}\sim\frac{A_e}{w}
$$

we obtain that the requirement (23) is met when an anisotropy angle $(\theta_1-\theta_e)$ is sufficiently small:

$$
\tan(\theta_1-\theta_e)=\frac{\varepsilon_a}{\varepsilon_\parallel}\frac{\tan\theta_1}{\left[1+\varepsilon_\perp/\varepsilon_\parallel\tan^2\theta_1\right]} << (k_e w)^{-1} \tag{24}
$$

The left-hand side of the inequality (24) reaches its maximal value

$$
\left[\tan(\theta_1-\theta_e)\right]_{max}=\frac{\varepsilon_a}{2\sqrt{\varepsilon_\perp\varepsilon_\parallel}}
$$

when

$$
\theta_1=\theta_1^{cr}=\arctan\left(\sqrt{\varepsilon_\parallel/\varepsilon_\perp}\right)
$$

Usually the optical anisotropy of uniaxial crystals is small: $(n_o - n_e) \sim 10^{-2}$,[17] and the inequality (24) is held for any angle θ_1 and any reasonable beamwidth w. Liquid crystals, however, possess a strong optical anisotropy,[2] and in this case the left-hand side of the inequality (24) may reach a considerable value for the angles close to θ_1^{cr}. Using the typical values of $\varepsilon_\perp = 2.3$, $\varepsilon_\parallel = 2.9$ [2] we find that

$$[\tan(\theta_1 - \theta_e)]_{max} = 0.12$$

and in such a case the self-focusing and self-trapping of an extraordinary beam with $k_e \sim 5 \cdot 10^4$ cm^{-1} are hardly possible for a realistic light intensity. When θ_1 is sufficiently small, the conditions (23) and (24) may be met, and the first term in the equation (22) may be ignored.

Omitting this term we obtain

$$\begin{aligned} & i\frac{\partial A_e}{\partial x''} + \frac{1}{2l_{e\parallel}}\frac{\partial^2 A_e}{\partial z''^2} + \frac{1}{2l_{e\parallel}}\left(\frac{\omega}{c}\right)^2 |A_e|^2 A_e \times \\ & \times h_e^2 \left[4\pi B \sin^2\theta_e \left(1 + \frac{\varepsilon_a}{\varepsilon_\perp} e_z \sin\theta_e\right)\right]^{-1} = 0 \end{aligned} \tag{25}$$

where

$$l_{e\parallel} = k_{e\parallel}\left(1 + \frac{\varepsilon_a}{\varepsilon_\perp} e_z^e \sin\theta_e\right)^{-1}$$

The effective nonlinear part n_{2eff}^e of the refractive index in an anisotropic medium may be evaluated as follows:

$$n_{2eff}^e = h_e^2 \left[8\pi B \sqrt{\varepsilon_\perp \varepsilon_\parallel} \sin^2\theta_e \left(1 + \frac{\varepsilon_a}{\varepsilon_\parallel} e_z^e \times \sin\theta_e\right)\right]^{-1} \tag{26}$$

The comparison of n_2^o and n_{2eff}^e shows that both quantities are of the same order of magnitude, but n_{2eff}^e strongly depends on the polarization and propagation direction of the extraordinary beam.

The relaxation time of the self-focusing process determined by the normal layer deformation is connected with the decay of the so-called second sound (SS) which is specific for SmA and represents the oscillations of the phase of the smectic order parameter.[5,14,15] The dissipation of SS is determined by SmA viscosity and the relaxation time τ may be in general evaluated as follows:

$$\tau \sim (\rho w^2)/\alpha_i$$

where the width of a spatial soliton w is used as a characteristic dimension of a perturbation instead of a SS wavelength, ρ is a mass density of SmA, and α_i is one of Leslie viscosity coefficients.[5] Using the typical values of the material parameters $\rho \sim 1$ g cm^{-3}, $\alpha_i \sim 1$ Poise,[5,14] and $w \sim 10^{-2}$ cm we obtain $\tau \sim 10^{-4}$ sec.

3. THE SELF-TRAPPING OF THE ORDINARY AND EXTRAORDINARY BEAMS IN SmA.

3.1. The Self-trapping in the Bulk of SmA.

The equations (15) and (21) together with the boundary conditions of the type (12) have the stable soliton-like solutions in the form of a homogeneous waveguide channel:[11–13]

$$A_o = |A_{o\,max}| \exp i \left\{ \frac{a_\perp^2 |A_{o\,max}|^2}{16\pi B \sqrt{\varepsilon_\perp}} \cdot \frac{\omega}{c} \cdot x' \right\} \times \operatorname{sech} \left\{ \frac{a_\perp |A_{o\,max}|}{2\sqrt{2\pi B}} \frac{\omega}{c} z' \right\} \tag{27}$$

$$A_e = |A_{e\,max}| \exp i \left\{ h_e^2 \left(\frac{\omega}{c}\right)^2 \frac{|A_{e\,max}|^2}{16\pi B l_{e\parallel} \sin^2\theta_e \left(1 + \frac{\varepsilon_a}{\varepsilon_\perp e_z^e \sin\theta_e}\right)} x'' \right\} \times \operatorname{sech} \left\{ \frac{h_e |A_{e\,max}| \omega}{2\sqrt{2\pi B} \sin\theta_e \cdot c \cdot \left(1 + \frac{\varepsilon_a}{\varepsilon_\perp} \sin\theta_e\right)^{\frac{1}{2}}} z'' \right\} \tag{28}$$

The widths $w_{o,e}$ of the spatial solitons (27) and (28) have the form:

$$w_o = \frac{2\sqrt{2\pi B}}{a_\perp |A_{o\,max}|} \left(\frac{\omega}{c}\right)^{-1} \tag{29}$$

$$w_e = \frac{2\sqrt{2\pi B} \sin\theta_e}{h_e |A_{e\,max}|} \left(\frac{\omega}{c}\right)^{-1} \left(1 + \frac{\varepsilon_a}{\varepsilon_\perp} e_z^e \sin\theta_e\right)^{\frac{1}{2}} \tag{30}$$

In the limiting case of the propagation direction parallel to the layer plane the spatial soliton of the amplitude of the ordinary beam (27) is distributed along the optical axis keeping its width the same as in the case of oblique propagation according to the expression (29).

The extraordinary beam propagating along the X axis is polarized parallel to the optical axis in such a way that

$$e_z^e = 1, \quad \theta_e = \pi/2, \quad k_e = k_{e\|} = \frac{\omega}{c}\sqrt{\varepsilon_\|}, \quad h_e = a_\|$$

Substituting these values into the relationship (30) we obtain

$$w_e = \frac{2\sqrt{2\pi B}}{a_\| |A_{e\,max}|}\left(\frac{\omega}{c}\right)^{-1}\sqrt{\frac{\varepsilon_\|}{\varepsilon_\perp}}$$

Comparing the solutions (27) and (28) one may see that in the case of an arbitrary polarized beam containing both an ordinary and an extraordinary components each of these components may propagate independently, if the tranverse distance between their axes is greater than $(w_o + w_e)/2$. The equations (3a, b), (11) and (17) yield the difference angle $\Delta\theta$ between the propagation directions of the two beams, i.e. between the wave vector $\vec{k}_o$ and the beam vector $\vec{s}$:

$$\Delta\theta = \arctan\left\{\frac{\varepsilon_\perp}{\varepsilon_\|}\frac{\sin\theta_o}{\left(1 - \frac{\varepsilon_\perp}{\varepsilon_\|}\sin^2\theta_o\right)^{1/2}}\right\} - \theta_o \tag{31}$$

The separation distance L_s has the form:

$$L_s \sim (w_e + w_o)/4\left|\sin\frac{\Delta\theta}{2}\right| \tag{32}$$

For the typical value of the optical anisotropy of a liquid crystal $(\varepsilon_a/\varepsilon_\perp \sim 0.3)$ [2] and for a comparatively large incidence angle

$$\theta_o > \pi/4$$

the difference angle reaches a considerable value, too: $\Delta\theta \sim (10^\circ - 20^\circ)$. For such a magnitude of $\Delta\theta$ the beams would separate after covering a distance comparable with their width which is much less than the characteristic length of the phase modulation L_f as was mentioned above:

$$L_f \cong kw^2 >> w$$

The power P of the slab-shaped beam[12] with the dimension in the Y direction $d_y >> w$ has the form:[11]

$$P = (cd_y/4\pi)\int_{-\infty}^{\infty}|A(R)|^2 dR \tag{33}$$

where R is the transverse coordinate (z' or z'' in our case). Substituting, for example, the relationship (27) into (33) we obtain:

$$P = 4c^3 B d_y / a_{\perp}^2 w \omega^2 \tag{34}$$

The critical power P_{cr} of self-trapping may be evaluated by using the well-known relationship:[1,12]

$$P_{cr} = (1.22\lambda)^2 c/64n_2$$

where λ is a light wavelength. Using the value of $n_2 \sim 3 \cdot 10^{-10}$ esu specific for the nonlinearity considered and $\lambda = 10^{-4}$ cm we find $P \sim 0.002$ MW which may be realized.[18] This power is two orders of magnitude lower than the one for electrostriction nonlinearity in carbon disulfide or benzene.[12] For this critical power and $d_y \sim 10^{-1}$ cm the equation (34) yields $w \sim 10^{-2}$ cm. The samples of SmA with a thickness of $(10^{-2} - 5 \cdot 10^{-2})$ cm are known.[14] This estimation is also valid for the extraordinary beam in the case of a sufficiently small anisotropy angle.

3.2. The Self-trapping of the Guided Wave at the Interface Between a Linear Medium and SmA.

Consider the light wave

$$E_y = A(z) \exp i(\beta x - \omega t) + c.c. \tag{35}$$

which is polarized parallel to the Y axis and propagates along the interface $z = 0$ between an isotropic homogeneous medium with the dielectric constant $\varepsilon_s (z < 0)$ and SmA cladding[19] $(z > 0)$. Here β is a propagation constant. The self-trapped solution in this case represents a bright surface wave:[19,20]

$$A(z) = A_{max} \operatorname{sech}\left(\frac{z - z_0}{w_o}\right), \qquad A_{max} = A(z_0) \tag{36}$$

The intensity maximum $|A_{max}|^2$, its position $z_0 > 0$, and the intensity $|A(0)|^2$ at the interface are determined by the boundary conditions which yield:[19]

$$\begin{aligned} A(0) &= A(z_0)\operatorname{sech}\left(z_0/w_o\right) \\ \varepsilon_s - \varepsilon_\perp &= \frac{a_\perp^2}{8\pi B}|A(0)|^2 \end{aligned} \tag{37}$$

It is seen from the equations (37) that the self-trapping of a guided wave is possible, if ε_s is slightly greater than $\varepsilon_\perp$. The propagation constant β may be found from the so-called β-power formula, binding it with the frequency ω and the power per unit length of wavefront P:[19]

$$\begin{aligned} P &= \left(c/4\pi\right)\int_{-\infty}^{\infty}|E_y|^2dz = \\ &= \frac{1}{8\pi}\frac{\beta c^2}{\omega}\left(\frac{a_\perp^2}{8\pi B}\right)^{-1}\Bigg\{(\varepsilon_s - \varepsilon_\perp)\times \\ &\times\left[\beta^2 - \varepsilon_s\left(\frac{\omega}{c}\right)^2\right]^{-\frac{1}{2}} + 2\left(\frac{\omega}{2}\right)^{-2}\times \\ &\times\left[\sqrt{\beta^2 - \varepsilon_s\left(\frac{\omega}{c}\right)^2} - \sqrt{\beta^2 - \varepsilon_\perp\left(\frac{\omega}{c}\right)^2}\right]\Bigg\} \end{aligned} \tag{38}$$

where the relationships (35)–(37) are used.

The equation (38) is a nonlinear dispersion relation for the surface wave (35).[19]

The layer displacement $u(z)$ may be easily calculated from the equation (13) using the expressions (35)–(37) and the condition of the absence of the displacement at the solid surface $z = 0$:

$$u(0) = 0$$

Namely, we obtain:

$$u(z) = \left(\frac{\omega}{c}\right)^{-1}\frac{A(z_0)}{\sqrt{2\pi B}}\left\{\tanh\left(\frac{z - z_0}{w_o}\right) + \tanh\left(\frac{z_0}{w_o}\right)\right\} \tag{39}$$

4. CONCLUSIONS.

A new mechanism of the Kerr nonlinearity caused by the normal layer deformation gives rise to the light beam self-focusing and self-trapping in SmA. The magnitude of the nonlinear part of the refraction index n_2 determined by this mechanism is about 10^{-10} esu, which is one or two orders of magnitude greater than the one for an orientational Kerr effect and an electrostriction in organic liquids.[1,12]

Comparison of the new type of nonlinearity with GON in NLC and SmA[2,4,7] shows that the light beam intensity applied in the case of the layer compression in SmA may be much greater than in the case of the purely orientational nonlinearity.

Both an ordinary and an extraordinary beams undergo self-focusing in SmA due to the layer compression. However, the self-focusing and the self-trapping of an extraordinary beam are possible only when the angle between its wave vector and beam vector caused by an optical anisotropy of SmA is sufficiently small.

When the light beam intensity surpasses a threshold value, an ordinary beam as well as an extraordinary one are self-trapped taking a form of a tranverse spatial soliton.[9,12,13] In the case of a sufficiently large incidence angle two self-trapped field components of an arbitrary polarized beam would be separated in space and would propagate independently in the essentially different directions due to the large optical anisotropy of SmA. The propagation of a bright surface guided wave[20] is possible at the interface between the linear medium and SmA when a dielectric constant of a linear medium is slightly greater than the transverse dielectric constant of SmA and when the light beam intensity satisfies the specific nonlinear dispersion relation known as the β-power formula.[19]

The numerical estimations show that for the self-trapping of a slab-shaped beam[12] with a width $w \sim 100\mu\text{m}$ a critical power $P \sim 0.002\,\text{MW}$ is required.

The mechanical instabilities in SmA caused by the normal deformation of layers have been investigated experimentally by means of the elastic light scattering.[5,21,22] Therefore one may expect that the nonlinear optical effects discussed above might be also observable, if the sufficiently strong pumping is used.

REFERENCES.

1. O. Zvelto, in Progress in Optics, edited by E. Wolf (North Holland, Amsterdam, 1974), vol. 12, Chap. 1, pp. 3, 4, 10, Chap. 2, pp. 14, 23, 24, Chap. 3, pp. 25, 26.
2. I. C. Khoo, in Progress in Optics, edited by E. Wolf (North Holland, Amsterdam, 1988), vol. 26, Chap. 1, pp. 107, 108, Chap. 2, pp. 115, 127, Chap. 3, pp. 133-144.

3. S. M. Arakelyan and Yu. S. Chilingaryan, Nonlinear Optics of Liquid Crystals (Nauka, Moscow, 1984) (in Russian), Chap.3 , pp. 112-116, Chap. 5, p. 208.
4. B. Ya. Zel'dovich and N. V. Tabiryan, Sov. Phys. Usp., 28, 1059 (1985).
5. P. G. de Gennes and J. Prost, The Physics of Liquid Crystals, Second Edition (Clarendon Press, Oxford, 1993), Chap. 1, pp. 10-12, 19, Chap. 3, pp. 102-104, Chap. 7, pp. 337-339, 370-372, Chap. 8, pp. 421-423.
6. Y. R. Shen, The Principles of Nonlinear Optics (John Wiley, New York, 1984), Chap. 3, pp. 42, 47, Chap. 16, pp. 286, 287, 290, Chap. 17, pp. 303-308.
7. N. V. Tabiryan and B. Ya. Zeldovich, Mol. Cryst. Liq. Cryst., 62, 637 (1981).
8. N. V. Tabiryan and B. Ya. Zeldovich, Mol. Cryst. Liq. Cryst., 69, 31 (1981).
9. Govind B. Agrawal, in Contemporary Nonlinear Optics, edited by Govind B. Agrawal and Robert Boyd, (Academic Press, New York, 1992), Chap. 2, pp. 76-78.
10. M. J. Stephen and J. P. Straley, Rev. Mod. Phys., 46, 617 (1974).
11. L. D. Landau and E. M. Lifshitz, Electrodynamics of Continuous Media, Second Edition, (Pergamon Press, Oxford, 1984), Chap. 2, p. 54, Chap. 9, pp. 257-260, Chap. 11, pp. 333-340.
12. R. Y. Chiao, E. Garmire and C. H. Townes, Phys. Rev. Lett., 13, 479 (1964).
13. V. E. Zakharov and A. B. Shabat, Sov. Phys. JETP, 34, 62 (1972).
14. L. Ricard and J. Prost, J. Physique (Paris), 42, 861 (1981).
15. P. G. de Gennes, J. Physique (Paris), Colloque C4, 30, C4-65 (1969).
16. A. P. Sukhorukov, The Nonlinear Interactions of Waves in Optics and Radiophysics (Nauka, Moscow, 1988) (in Russian), Chap. 1, pp. 15-16.
17. V. G. Dmitriev, G. G. Gurzadyan and D. N. Nikogosyan, Handbook of Nonlinear Optical Crystals (Springer Verlag, 1991), Chap. 3, pp. 74-94.
18. J. C. A. Chaimowicz, Optoelectronics. An Introduction (Butterworth-Heinemann, Oxford, 1989), Chap. 9.6, p. 158.
19. D. Michalache, M. Bertolotti and C. Sibilia, in Progress in Optics, edited by E. Wolf (North Holland, Amsterdam, 1989), vol. 27, Chap. 2, pp. 233-235.
20. Yijing Chen, Phys. Rev. A, 45, 4974 (1992).
21. H. Birecki, R. Shaetzing, F. Rondelez, and J. D. Litster, Phys. Rev. Lett., 36, 1376 (1976).
22. Noel A. Clark, Phys. Rev. A, 14, 1554 (1976).